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Counterparty effects on capital structure decision in incomplete market

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ABSTRACT

This paper builds a static contingent-claim model that allows for examining the optimal capital structure with the joint arguments of counterparty default risk and market incompleteness. A first-passage-time model with jump default barrier is adopted to capture the counterparty effects on the pricing of defaultable claims. Following the framework of Jarrow and Yu (2001), the jump in primary firm's bankruptcy barrier is designed as the loss on capital resulted from secondary firm's bankruptcy. The relevance of market incompleteness in the context of claim-pricing is considered using "good-deal asset price bound" method by Cochrane and Saa-Requejo (2000). We show that the effects of counterparty's default clearly diminish the uses of debt, which indirectly explains the so-called under-leveraged puzzle. We further find that counterparty effects on capital structure are sensitive to market incompleteness and firm's characteristics, such as tax rate and bankruptcy cost rate.

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1. Introduction

Since the seminar work by Modigliani and Miller (1958), there is a vast literature on corporate finance that devotes to explaining the central anomaly related to firm's financing decision: low leverage-use (so-called underleveraged puzzle). The issues on such a subject explored by prior research include agency costs (Jensen and Meckling, 1976; Leland, 1998; Myers, 1977), information asymmetries (Myers and Majluf, 1984), asset illiquidity (Morellec, 2001), empirical test (Ju et al., 2005), and macro-economic risk (Chen, 2010; Hackbarth et al., 2006). A consensus view underlying these works is that bankruptcy costs derived from standard structural model are too small to offset the value of tax shields, and thus, other cost-factors must be introduced into trade-off analysis to explain observed capital structures. Motivated by this idea, the paper intends to address the central question on capital structure theory from the arguments of counterparty defaults.

The reason of why this study puts a special focus on counterparty's default risk is sketched as follows. In view of credit contagion and clustering of default during the credit crises, many of the studies hold that the standard credit models without considering counterparty effects (e.g., Merton, 1974) will under-predict a firm's default risk and mis-estimate the value of derivatives on defaultable assets, such as credit default swap. To overcome this modeling restriction, a class of works on the counterparty risk has been motivated. Jarrow and Yu (2001) pioneer in pricing the default-risky securities with counterparty risk using reduced-form model, proposed by Jarrow and Turnbull (1995). Except for Jarrow and Yu, subsequent works on the pricing of credit default swap that particularly focus on the risk accompanied by counterparty defaults include Brigo and Tarenghi (2005), Turnbull (2005), Walker (2006), Duffie and Zhu (2009), and Leung and Kwok (2005, 2009). Kraft and Steffensen (2007) develop a unifying framework to capture credit contagion. Thompson (2007) and Chang and Yu (2009) make the application on insurance contract pricing. Jorion and Zhang (2009) offer the empirical evidence on credit contagion via direct counterparty effects. The above-mentioned literatures provide an important implication that the effects of counterparty's defaults are crucial for measuring the credit risk. Because of a strong linkage between firm's leverage and default risk, this paper thus develops a capital structure model combined with the impacts of counterparty's default.

It is noteworthy that the existing counterparty-related credit models mainly are of intensity-based approach. These models cannot be applied in capital structure problem due to the underlying assumption that the default is irrelevant with firm's value and capital structure. For this reason, the modeling spirit of paper must follow firm-value based approach. However, substantial empirical tests have found the evidence of asset illiquidity (see, e.g., Strebulaev, 2003; Gibson and Mougeot, 2004; Martineza et al., 2005; Eisfeldt, 2007; Khandani and Lo, 2009). In addition, for levered firms, the free disposition of assets will likely be restricted by their bond protective

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covenants (see, Morellec, 2001). If the asset non-tradability or illiquidity does exist, it is impossible to perfectly replicate the focus payoff associated with claims on underlying assets using the continuous trading strategies. In other words, no-arbitrage based pricing argument in a continuous-trading economy fails, and the complete market-assumption becomes inappropriate. To overcome this problem, the paper adopts Cochrane and Saa-Requejo's (2000) "good-deal" price bound method when modeling the dynamic of asset value. Such method is featured by "stochastic discount factor". Imposing relevant constrains on the discount factor, the pricing of uncertain payoff can be undertaken in incomplete market or situations where the purely preference-free approach suffers the breakdown; such as thin trading or non-tradable circumstances (see Section 2.1 for more detailed introduction).

This paper captures the counterparty effects by establishing a first-passage-time model with jump default barrier under primary–secondary framework. Specifically, an upward jump accompanied by counterparty's default is imposed on the primary firm's bankruptcy-triggering threshold. Based on this framework, the paper develops a static contingent claim model for primary firm's capital structure decision. The model considers a common case where the primary firm (like parent company) may offer the secondary firm (like subsidiary) a credit guarantee against default promising their collateral demands for defending joint interests. Once the secondary firm bankrupts, the obligation of primary firm will be expanded to lead to a higher bankrupt potential. The optimization of primary firm's capital structure here is determined by the tradeoff between traditional levered benefits (i.e., tax shields minus bankruptcy costs) and the capital loss in counterparty's default. The values of contingent claims involved in the financing decision will be derived using the martingale method.

The remainder of paper is structured as follows. Section 2 makes a brief review to "good-deal" price bound method, and builds first-passage-time model with jump default barrier. Section 3 extends Section 2 to make an application on capital structure theory. The numerical analysis is given in Section 4. Lastly, Section 5 concludes.

2. Preliminaries

In this section we firstly make a review to Cochrane and Saa-Requejo's (2000) method particularly for asset pricing in incomplete markets, and then establish a first-passage-time model with jump default barrier under primary–secondary framework.

2.1. A review of "good-deal" asset pricing bound

For a levered firm, the assets usually are illiquid or non-traded, because the debt covenants may restrict the disposition of assets (see, Morellec, 2001). As a result, the payoffs associated with claims on these types of assets cannot be perfectly replicated. Thus the traditional no-arbitrage arguments are not applied in this case. To address the problem, Cochrane and Saa-Requejo (2000) propose "good-deal" asset pricing bound method. They show that, in incomplete market, the claims on illiquid assets will be traded at a range of price, rather than single price. The tightness of price bound relies on the degree of underlying assets' illiquidity. We now give a brief introduction to the theory of illiquid-asset pricing.

Let x be a focus payoff associated with contingent claim on reference firm's illiquid assets V, and Λ be the stochastic discount factor with a fixed upper bound on its volatility $\mathcal{A}^2>0$. Following Cochrane and Saa-Requejo (2000) and Hung and Liu (2005), to exclude high Sharpe ratio and arbitrage opportunities, some restrictions need to be imposed on the discount factor. Imposing the positive, volatility, and white noise constrains on the unique discount factor and trying all kinds of combinations of non-binding and binding constrains, this yields the lower good-deal price bound for contingent claim issued by reference firm, $\underline{C}_x$, shown by the following equation:

$$\underline{C}_x(0) := \min_{\Lambda} \mathrm{E_P}\left(\int_0^\infty x(t,V(t))\Lambda(t)/\Lambda(0)dt\,|\,\mathcal{H}_0\right) \equiv \mathrm{E_P}\left(\int_0^\infty x(t,V(t))\underline{\Lambda}(t)/\underline{\Lambda}(0)dt\,|\,\mathcal{H}_0\right) \tag{2.1}$$

$$s.t. V(0) = \mathrm{E_P}(V(t)\Lambda(t)/\Lambda(0)\,|\,\mathcal{H}_0);\ \Lambda(t)>0;\ dt^{-1}\mathrm{E_P}\left((d\Lambda(t)/\Lambda(0))^2\,|\,\mathcal{H}_0\right)\le\mathcal{A}^2;$$

$$m\times Cov\left(d\Lambda(t)/\Lambda(t), dB_A^{P'}(t)\right) =Ov\left(d\Lambda(t)/\Lambda(t), dB_B^{P'}(t)\right)^{1};\ \forall t\in[0,\infty],$$

where $\mathrm{E_P}(\cdot)$ is the expectation operator under physical measure P; $\mathcal{H}_0$ denotes the collection of information on the white noises dB_B^P and dB_A^P; and $m\in[0,\infty]$ is a constant parameter. Similarly, the upper good-deal bound $\overline{C}_x$ can be derived from the corresponding maximum.

The good-deal price bound method is useful in modeling the financing decision of a firm with illiquid or non-traded assets. It helps us derive the value of contingent claims involved with capital structure problem, such as tax benefits, bankruptcy costs, firm's debt, and equity. The optimization of leverage is achieved by choosing a debt level that maximizes firm's total value.

2.2. A first-passage-time model with jump default barrier

Consider a primary–secondary framework in the continuous-time economy. For brevity, let firm A play the primary firm and B the secondary firm. Assume that there is a complete probability space (Ω, $\mathcal{F}$, P), rich enough to support the value processes of non-traded firm i assets $V^i=\{V^i(t);\ 0\le t<\infty\}$ and corresponding twin securities $S^i=\{S^i(t);\ 0\le t<\infty\}$ for $i=A, B$.[2]

Also assume that there is a finite time span $[0, T]$, where $T\in\Re^+$ is valid for both firm A and B. Denote by $\tau^i:\Omega\to\Re^+, i=A,B$ the first passage F-stopping times for firms A and B respectively. Each random time means the first moment in the finite interval $[0, T]$ that the underlying assets' value falls below its predetermined bankruptcy triggering threshold and can be defined as

$$\tau^A := \inf\left(t\in[0,T]: V^A(t)\le F^A + 1_{(\tau^B\le t)}d\right), \tag{2.2}$$

$$\tau^B := \inf\left(t\in[0,T]: V^B(t)\le F^B\right), \tag{2.3}$$

with the convention: $\inf\phi=+\infty$. In Eqs. (2.2) and (2.3), $F^i>0, i=A, B$ can be served as debt's par value of firm i. The jump term d is assumed to be a predetermined fraction of B's promised repayment, reflecting an increment in A's debt obligation accompanied by B's default. The size of jump interprets the intensity of counterparty risk. A natural implication behind jump default barrier is: since A may use its assets as the collaterals for B's liabilities to defend common interests, B's default will transfer a portion of its own obligation into A. And the likelihood of A's default will be greater after B goes bankrupt.

The dynamics under this real measure P of non-traded firm i assets' value and of the price of corresponding tradable twin security are given by

$$dV^i(t)/V^i(t) = \left(\mu_V^i-\delta_V^i\right)dt + \sigma_{V_Z^i}dZ_i^P(t) + \sigma_{V_W^i}d\ W_i^P(t) \tag{2.4}$$

and

$$dS^i(t)/S^i(t) = \left(\mu_S^i-\delta_S^i\right)dt + \sigma_{S^i}\ dZ_i^P(t), i=A,B \tag{2.5}$$

[1] Hung and Liu (2005) impose this constraint on discount factor to assume that the impact of white noise dB_B^P on discount factor is m times greater than white noise dB_A^P.

[2] Cochrane and Saa-Requejo (2000) state that, once the purely preference-free approach breaks down, one may be unable to trade continuously and to achieve the perfect hedge. If so, there still exists a tradable asset in the market that can be used for approximately hedging non-traded underlying asset. Such assets used for approximate hedge are termed "twin securities" in Hung and Liu (2005).

where μ_V^i, μ_S^i, δ_V^i, δ_S^i, $\sigma_{V_Z^i}$, $\sigma_{V_W^i}$ and σ_{S^i} are all fixed and positive parameters, respectively representing appreciation rate, asset payout ratio, and volatility for assets value. $W_i^P=\{W_i^P(t);\ t\in[0,\infty)\}$ and $Z_i^P=\{Z_i^P(t);\ t\in[0,\infty)\}$ both follow standard single-dimensional Brownian motion with respect to $F_t^{iW}=\sigma(W_i^P(u),\ 0\le u\le t)$ and $F_t^{iZ}=\sigma(Z_i^P(u),\ 0\le u\le t)$ respectively. The joint filtration governing the randomness of firm i assets' value has a specified definition as $\mathcal{F}^i=(F_t^i)_{t\ge 0}:=\mathcal{F}^{iW}\vee\mathcal{F}^{iZ}$ where $F_t^i:=F_t^{iW}\vee F_t^{iZ}$, $\mathcal{F}^{iW}=(F_t^{iW})_{t\ge 0}$, and $\mathcal{F}^{iZ}=(F_t^{iZ})_{t\ge 0}$. The original filtration F thus satisfies $\mathcal{F}=(F_t)_{t\ge 0}:=\mathcal{F}^{\mathcal{A}}\vee\mathcal{F}^{\mathcal{B}}$.

The relationships among the noise in Eqs. (2.4) and (2.5) are summarized as:

$$\rho_{Z_AW_A}dt=\mathrm{E_P}\left(dZ_A^P(t)dW_A^P(t)\right)=0,\ \rho_{Z_AW_B}dt=\mathrm{E_P}\left(dZ_A^P(t)dW_B^P(t)\right)=0,$$
$$\rho_{Z_BW_A}dt=\mathrm{E_P}\left(dZ_B^P(t)dW_A^P(t)\right)=0,\ \rho_{Z_BW_B}dt=\mathrm{E_P}\left(dZ_B^P(t)dW_B^P(t)\right)=0.$$

These determine the correlation coefficient between the dynamic value of firm i assets and its twin security; i.e., $\rho_i=\sigma_{V_Z^i}/(\sigma_{V_Z^i}^2+\sigma_{V_W^i}^2)^{0.5}$. Since it is impossible for perfect hedge in incomplete market, non-traded asset cannot be perfectly replicated from its twin security. This helps us explain why W_i^P and Z_i^P are uncorrelated. When the replication is less than perfect, the information for exactly determining the assets' value is not enough and the law of single price breaks down.

The dynamic of stochastic discount factor has an explicit expression as:

$$d\Lambda(t)/\Lambda(t)=-rdt-h_S^A d\,Z_A^P(t)-h_S^B d\,Z_B^P(t)+\gamma^A dW_A^P(t)+\gamma^B dW_B^P(t) \tag{2.6}$$

where $h_S^A:=(\mu_S^A-r)\ \sigma_{S^A}{}^{-1}$, $h_S^B:=(\mu_S^B-r)\ \sigma_{S^B}{}^{-1}$, $\gamma^A:=a((\mathcal{A}^2-(h_S^A)^2-(h_S^B)^2)/(m+1))^{0.5}$, and $\gamma^B:=m^{0.5}\gamma^A$. In Eq. (2.6)$r$ represents the constant risk-free interest and $a\in[-1,1]$ determines the bound for discount factor; e.g., $min\Lambda=\underline{\Lambda}$ for $a=-1$. Identical with Eqs. (2.4) and (2.5), all usual conditions are satisfied by Eq. (2.6), and hence there exists a unique, strong, global solution to this SDE. Using Ito's lemma, we have, for $t\in[0,T]$,

$$\Lambda(t)=\Lambda(0)\exp\left\{\int_0^t\left(-r-0.5\ a^2A^2-0.5\left(1-a^2\right)\left(\left(h_S^A\right)^2+\left(h_S^B\right)^2\right)\right)du\right.$$
$$\left.-\int_0^t h_S^A\,d\,Z_A^P(u)-\int_0^t h_S^B dZ_B^P(u)+\int_0^t\gamma^A dW_A^P(u)+\int_0^t\gamma^B dW_B^P(u)\right\} \tag{2.7}$$

To solve the random times as Eqs. (2.2) and (2.3), it is essential to introduce a specific probability measure R equivalent to P on $(\Omega,\mathcal{F})$ by setting, for $t\in[0,T]$,

$$\xi(t):=\left.\frac{d\mathrm{R}}{d\mathrm{P}}\right|_{F_t},\mathrm{P}-\text{a.s.}$$

Thus this $\mathcal{F}$-martingale Radon–Nikodym density process under P has an integral representation as the following:

$$\xi(t)=1+\int_0^t\xi(u)\left(-h_S^A\ dZ_A^P(u)-h_S^B\ dZ_B^P(u)+\gamma^A\ dW_A^P(u)+\gamma^B\ dW_B^P(u)\right)=\Lambda(t)e^{rt}, \tag{2.8}$$

which ensures that the properties of the measure R can be fitted. We now describe the application of Girsanov's theorem on the measure change.

Theorem 2.1. Given a probability measure R equivalent to P on $(\Omega,\mathcal{F})$ and the Radon–Nikodym density process ξ with respect to P shown by Eq.(2.8), the processes, for $i=A,B$,

$$Z_i^R(t)=Z_i^P(t)+\int_0^t h_S^i\ du,W_i^R(t)=W_i^P(t)-\int_0^t\gamma^i du,\quad\forall t\in[0,T],$$

respectively follow a single-dimensional $\mathcal{F}^{iZ}$- and $\mathcal{F}^{iW}$-Brownian motion under R. Further, the relationships between these processes are all mutually orthogonal.

Based on Theorem 2.1, the dynamics of firm i assets' value can be rewritten as:

$$\begin{aligned}dV^i(t)/V^i(t)&=\left(\mu_V^i-\delta_V^i-h_S^i\sigma_{V_Z^i}+\gamma^i\ \sigma_{V_W^i}\right)dt+\sigma_{V_Z^i}d\ Z_i^R(t)+\sigma_{V_W^i}dW_i^R(t)\\&=\left(\left(\mu_V^i-\delta_V^i\right)\sigma_{V^i}^{-1}-h_S^i\rho_i+\gamma^i\left(1-\rho_i^2\right)^{0.5}\right)\sigma_{V^i}dt+\sigma_{V^i}d\mathcal{B}_i^R(t)\\&:=\eta^i dt+\sigma_{V^i}d\mathcal{B}_i^R(t),\end{aligned} \tag{2.9}$$

where $\mathcal{B}_i^R$ follows a single-dimensional Brownian motion under R with respect to $\mathcal{F}^i:=\mathcal{F}^{iW}\vee\mathcal{F}^{iZ}$; $\sigma_{V^i}:=(\sigma_{V_Z^i}{}^2+\sigma_{V_W^i}{}^2)^{0.5}$ and $\eta^i:=((\mu_V^i-\delta_V^i)\sigma_{V^i}{}^{-1}-h_S^i\rho_i+\gamma^i(1-\rho_i^2)^{0.5})\ \sigma_{V^i}$ respectively denote the aggregate diffusion and drift of V^i. Using Eq. (2.9) with the law of first-passage-time and Bayesian formula one can solve the probability functions for random times Eqs. (2.2) and (2.3) easily.

Theorem 2.2. Given the definition of random times as Eqs. (2.2) and (2.3), total probabilities of survival until time t on $\mathcal{F}_0=\mathcal{F}_0^{\mathcal{A}}\vee\mathcal{F}_0^{\mathcal{B}}$ under R are as follows. For $0\le t\le T$,

$$\begin{aligned}\mathrm{P_R}(\tau^A\ N\ t\,|\,\mathcal{F}_0)&\equiv F_{\tau^A}(t\,|\,\mathcal{F}_0)\\&=\left(\mathrm{N}[n_1(t)]-e^{-a\ x_A}\mathrm{N}[n_2(t)]\right)\left(\mathrm{N}[n_5(t)]-e^{-b\ x_B}\mathrm{N}[n_6(t)]\right)\\&\quad+\int_0^t\int_{F^A+d}^{\infty}\int_{F^A}^{x}\left(\mathrm{N}[n_3(t,u,x)]-e^{x_Ac(x)}\mathrm{N}[n_4(t,u,x)]\right)f_{\tau^B}(u\,|\,\mathcal{F}_0)\\&\quad\times f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{u\le T\})dy\ \ dx\ \ du\end{aligned}$$

and

$$\mathrm{P_R}(\tau^B>t\,|\,\mathcal{F}_0)\equiv F_{\tau^B}(t\,|\,\mathcal{F}_0)=\mathrm{N}[n_5(t)]-e^{-bx_B}\mathrm{N}[n_6(t)]$$

where

$$a=\begin{cases}+1 & \text{upper bound}\\-1 & \text{lower bound}\end{cases}$$
$$\lambda_A=\eta^A-0.5\ \sigma_{V^A}^2;\ \lambda_B=\eta^B-0.5\ \sigma_{V^B}^2;\ x_A=2\ \sigma_{V^A}^{-2}\ \lambda_A;\ x_B=2\ \sigma_{V^B}^{-2}\ \lambda_B;$$
$$a=\ln\left(V^A(0)/F^A\right);\ b=\ln\left(V^B(0)/F^B\right);\ c(x)=\ln\left(\left(F^A+d\right)/x\right);$$
$$n_1(t)=(a+\lambda_At)(\sigma_{V^A}\sqrt{t})^{-1};\ n_2(t)=(-a+\lambda_A\ t)(\sigma_{V^A}\sqrt{t})^{-1};$$
$$n_3(t,u,x)=(-c(x)+\lambda_A(t-u))(\sigma_{V^A}\sqrt{t-u})^{-1};$$
$$n_4(t,u,x)=(c(x)+\lambda_A(t-u))(\sigma_{V^A}\sqrt{t-u})^{-1};$$
$$n_5(t)=(b+\lambda_Bt)(\sigma_{V^B}\sqrt{t})^{-1};\ n_6(t)=(-b+\lambda_Bt)(\sigma_{V^B}\sqrt{t})^{-1};$$
$$f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_t\vee\{u\le t\})=\exp\left\{\left[2\ln y-\ln x-\ln V^A(0)+\lambda_A\ u\right]^2\left(-2\ \sigma_{V^A}^2u\right)^{-1}\right\}$$
$$\times\sqrt{2}\ln\left(xV^A(0)/y^2\right)\left(\sqrt{\pi}\ u^{3/2}\sigma_{V^A}^3x\ y)\right)^{-1}\left[y/V^A(0)\right]^{x_A}$$

is the joint density of $\inf_{0\le s<u}V^A(s)=y$ and $V^A(u)=x$ on $\mathcal{F}_0\vee\mathcal{J}_t\vee\{\tau^B=u\le t\}$ under R;

$$f_{\tau^B}(u\,|\,\mathcal{F}_0)=\left(e^{-b\,x_B}\mathrm{n}[n_6(u)]\left(\lambda_B+u^{-1}b\right)-\mathrm{n}[n_5(u)]\left(\lambda_B-u^{-1}b\right)\right)(2\ \sigma_{V^B}\sqrt{u}))^{-1}$$

is the density function of $\tau^B=u$ under R on F_0; $\mathrm{N}(\cdot)$ is the cumulative standard normal distribution; and $\mathrm{n}(\cdot)$is the standard normal density function.

The formulas given by Theorem 2.2 are useful tools for examining the properties of counterparty effects on firm's default and the value of associated contingent claims. We will repeatedly apply the above formulas to develop our main model in the next section. For detailed proofs of Theorem 2.2, see Appendix A.

3. Main model

This section rebuilds contingent claim model for capital structure with counter-party risk when considering that both primary and

secondary firms' assets are illiquid or non-traded. To introduce the economic setup, some relevant assumptions should be imposed.

3.1. Economic setup

Without loss of generality, consider a circumstance where both firms A and B are totally equity-financed at initial time-0, and intend to issue a debt with finite maturity T at the value equaling to its face value, meaning that $C^i_{Debt}(0)=F^i$ for $i=A, B$. For defending their common interests that are unaffected by capital structure, A may guarantee B against bankruptcy. Once B defaults, the debt obligation with respect to his creditors will be partially transferred into A, $d=F^B r_d$. Thus A uses a portion of his assets as the collateral for liquidating B's liabilities. The assets of firms A and B respectively has the current value $V^A(0)$ and $V^B(0)$. The value dynamics of their assets and twin securities follow the expressions as Eqs. (2.4) and (2.5).

Since this paper has a special focus on counterparty effects, B's capital structure will be treated as an exogenous variable during the analysis. Therefore, here we only give the introduction to A's debt contract. The debt continuously pays the coupon at a constant rate, $c^A>0$, which shields income from its taxes at the effective rate β. The level of coupon is jointly determined by sell-at-par condition $C^A_{Debt}(0)=F^A$ and discount factor; e.g., the lower bound of coupon $c^A=\underline{c}^A$, when $a=-1$ and $\Lambda=\underline{\Lambda}$. The debt has a protective covenant that specifies the condition of bankruptcy declaration. This follows the definition of random time as Eq. (2.2), meaning that, if the total value of its own assets falls to the predetermined threshold at any time during the life of the debt, A becomes insolvent and is forced into bankruptcy. Upon bankruptcy, A's equity becomes valueless and only a portion $1-\alpha^A$ of the value of remaining assets can be redeemed by debt-holders, where $\alpha^A\in[0,1]$ is the bankruptcy cost parameter. The fraction loss of remaining assets' value is expended in the bankruptcy process.

The tradeoff between debt's tax interests and bankruptcy loss helps A's managers make the optimal capital structure choice that maximizes firm's total levered value $TV^A(0)$. Following Ju (2001), assume that each agent in the economy has a utility on his own initial total wealth. Hence, the value-maximizing objective naturally ensures the utility-maximization. We are now ready to value A's contingent claims, including firm's debt, tax benefits, bankruptcy costs, and firm's equity.

3.2. The value of primary firm's debt

Following Ju et al. (2005), the current value of firm A's debt $C^A_{Debt}(0)$ sums a contribution from the coupon, a contribution from the recovered payment to debt holders if it bankrupts, and the repayment of the par if bankruptcy does not occur until maturity. That is, normalizing the focus payoff of problem (2.1) and letting $a=-1$,

$$\underline{C}^A_{Debt}(0) := \mathrm{E_P}\Big(\int_0^T \underline{c}^A\underline{\Lambda}(t)1_{(\tau^A>t)}dt + \left(1-\alpha^A\right)V^A\left(\tau^A\right)\underline{\Lambda}\left(\tau^A\right)\ 1_{(\tau^A\le T)} + F^A\underline{\Lambda}(T)1_{(\tau^A>T)}\Big|\mathcal{F}_0\Big)/\underline{\Lambda}(0) \tag{3.1}$$

To show the general representation, rewriting Eq. (3.1) as:

$$C^A_{Debt}(0) = \mathrm{E_P}\Big(\int_0^T c^A\Lambda(t)1_{(\tau^A>t)}dt + \left(1-\alpha^A\right)V^A\left(\tau^A\right)\Lambda\left(\tau^A\right)1_{(\tau^A\le T)} + F^A\Lambda(T)1_{(\tau^A>T)}\Big|\mathcal{F}_0\Big)/\Lambda(0) \tag{3.2}$$

Using Fubini's theorem and Eq. (2.8), the conditional expectation in Eq. (3.2) becomes

$$\begin{aligned} C^A_{Debt}(0) = \xi(0)^{-1}\Big(&\int_0^T c^A\mathrm{E_P}\left(\xi(t)\mathrm{e}^{-rt}1_{(\tau^A\ N\ t)}\,|\,\mathcal{F}_0\right)dt \\ &+ F^A\ \mathrm{E_P}\left(\xi(T)\mathrm{e}^{-rT}1_{(\tau^A\ N\ T)}\,|\,\mathcal{F}_0\right) \\ &+ \left(1-\alpha^A\right)\ \mathrm{E_P}\left(V^A\left(\tau^A\right)\xi\left(\tau^A\right)\mathrm{e}^{-r\tau^A}1_{(\tau^A\le T)}\,|\,\mathcal{F}_0\right)\Big). \end{aligned} \tag{3.3}$$

Based on Theorem 2.1, the measure underlying Eq. (3.3) is changed from P to R. Thus we have

$$\begin{aligned} C^A_{Debt}(0) = &\int_0^T c^A\ \mathrm{E_R}\left(\mathrm{e}^{-rt}1_{(\tau^A>t)}\,|\,\mathcal{F}_0\right)dt + F^A\mathrm{E_R}\left(\mathrm{e}^{-rT}1_{(\tau^A>T)}\,|\,\mathcal{F}_0\right) \\ &+ \left(1-\alpha^A\right)\mathrm{E_R}\left(V^A(\tau^A)\mathrm{e}^{-r\tau^A}1_{(\tau^A\le T)}\,|\,\mathcal{F}_0\right). \end{aligned}$$

To display the explicit solution of $C^A_{Debt}(0)$, we give the following proposition.

Proposition 3.1. Given initial condition of non-default and Theorem 2.2, the value of firm A's debt with finite maturity T, constant coupon payment c^A, and recovery ratio $1-\alpha^A$ consists of three parts: (i) the part of coupon

$$\begin{aligned} C1^A_{Debt}(0) &:= \int_0^T c^A\mathrm{E_R}\left(\mathrm{e}^{-rt}1_{((\tau^A>t))}\,|\,\mathcal{F}_0\right)dt \\ &= \int_0^T c^A\mathrm{e}^{-rt}\left(\mathrm{N}[n_1(t)]-e^{-ax_A}\mathrm{N}[n_2(t)]\right)\left(\mathrm{N}[n_5(t)]-e^{-bx_B}\mathrm{N}[n_6(t)]\right)dt \\ &\quad + \int_0^T\int_0^t\int_{F^A+d}^{\infty}\int_{F^A}^{x} c^A\mathrm{e}^{-rt}\left(\mathrm{N}[n_3(t,u,x)]-e^{x_Ac(x)}\mathrm{N}[n_4(t,u,x)]\right)f_{\tau^B}(u\,|\,\mathcal{F}_0) \\ &\quad \times f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{u\le T\})dx\ du\ dt \end{aligned}$$

(ii) the part of repayments of the principal

$$\begin{aligned} C2^A_{Debt}(0) &:= F^A\mathrm{E_R}\left(\mathrm{e}^{-rT}1_{(\tau^A>T)}\,|\,\mathcal{F}_0\right) \\ &= F^A\mathrm{e}^{-rT}\Big(\left(\mathrm{N}[n_1(T)]-e^{-ax_A}\mathrm{N}[n_2(T)]\right)\left(\mathrm{N}[n_5(T)]-e^{-bx_B}\mathrm{N}[n_6(T)]\right) \\ &\quad + \int_0^t\int_{F^A+d}^{\infty}\int_{F^A}^{x}\left(\mathrm{N}[n_3(T,u,x)]-e^{x_Ac(x)}\mathrm{N}[n_4(T,u,x)]\right)f_{\tau^B}(u\,|\,\mathcal{F}_0) \\ &\quad \times f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{u\le T\})dy\ dx\ du\Big) \end{aligned}$$

and (iii) the part of recovered payments

$$\begin{aligned} C3^A_{Debt}(0) &:= \left(1-\alpha^A\right)\mathrm{E_R}\left(V^A(\tau^A)\mathrm{e}^{-r\tau^A}1_{(\tau^A\le T)}\,|\,\mathcal{F}_0\right) \\ &= \Big(\int_0^T\int_{F^A+d}^{\infty}\int_{F^A}^{x}\int_{t_+}^{T} d\ \mathrm{e}^{-rs}f_{\tau^A}\left(s\,|\,\mathcal{F}_t^{\mathcal{A}}\vee\mathcal{F}_0^{\mathcal{B}}\vee\mathcal{J}_T\vee\left\{V^A(t)=x\right\}\right) \\ &\quad \times f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{t\le T\})f_{\tau^B}(t\,|\,\mathcal{F}_0)ds\ dy\ dx\ dt \\ &\quad + \int_0^T\int_{F^A}^{F^A+d}\int_{F^A}^{x}\left(x-F^A\right)f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{t\le T\})f_{\tau^B}(t\,|\,\mathcal{F}_0)\mathrm{e}^{-rt}dy\ dx\ dt \\ &\quad + \int_0^T F^Af_{\tau^A}(t\,|\,\mathcal{F}_0)\mathrm{e}^{-rt}dt\Big)\left(1-\alpha^A\right) \end{aligned}$$

where $f_{\tau^A}(\tau|\mathcal{F}_0)=-\partial\ F_{\tau^A}(t|\mathcal{F}_0)/\partial t$ denotes the density function of τ^A conditioned on $\mathcal{F}_0$ and

$$\begin{aligned} f_{\tau^A}\left(s\,|\,\mathcal{F}_t^{\mathcal{A}}\vee\mathcal{F}_0^{\mathcal{B}}\vee\mathcal{J}_T\vee\left\{V^A(t)=x\right\}\right) = &\left(\mathrm{e}^{x_Ac(x)}\mathrm{n}[n_4(s,t,x)]\left(\lambda_A-(s-t)^{-1}c(x)\right)\right) \\ &-\left(\lambda_A+(s-t)^{-1}c(x)\right)\mathrm{n}[n_3(s,t,x)])\left(2\sigma_{V^A}\sqrt{s-t}\right)^{-1} \end{aligned}$$

denotes the density function of $\tau^A=s>t$ conditioned on $\mathcal{F}_t^{\mathcal{A}}\vee\mathcal{F}_0^{\mathcal{B}}\vee\mathcal{J}_T\vee\{V^A(t)=x\}$. The sum of $C1^A_{Debt}(0)$, $C2^A_{Debt}(0)$, and $C3^A_{Debt}(0)$ equals the market value of debt $C^A_{Debt}(0)$.

Via Proposition 3.1, the value of debt, yield spreads over risk-free interest rate, and firm's default probabilities can be calculated. These financial variables contribute to the clear understanding of a firm's credit quality or financial health with unilateral counterparty risk in incomplete market. Further, the proposed formula can be applied in deriving the value of debt's tax benefits and bankruptcy costs. These will be shown in the following subsection.

3.3. The value of primary firm's tax benefits, bankruptcy costs, and equity

The tax benefits enjoyed by debt financing can be interpreted as a security which pays a constant coupon rate equal to the tax-sheltering value of the interest payments as long as the firm is solvent. Otherwise, after bankruptcy, tax benefits cannot be claimed. The lower good-deal bound for present value of firm A's tax shield has the representation

$$\underline{C}^A_{TB}(0) = \underline{\Lambda}(0)^{-1}\mathrm{E_P}\left(\int_0^T\beta\underline{C}^A\underline{\Lambda}(t)1_{(\tau^A>t)}dt\,|\,\mathcal{F}_0\right) = \beta\underline{C1}^A_{Debt}(0).$$

The bankruptcy costs, as implied by the name, means that the fraction of firm assets' value may be lost in bankrupt negotiation or firm reorganization at the moment of default. Following our assumption, it is known that bankruptcy process consumes $\alpha^A V^A(\tau^A)$. The lower bound of the value of bankruptcy costs at initial time is

$$\underline{C}^A_{BC}(0) = \underline{\Lambda}(0)^{-1}\mathrm{E}_{\mathrm{P}}\left(\alpha^A V^A\left(\tau^A\right)\underline{\Lambda}\left(\tau^A\right)1_{(\tau^A\le T)}\,|\,\mathcal{F}_0\right) = \alpha^A \underline{C3}^A_{Debt}(0)\,/\left(1-\alpha^A\right).$$

With similar derivation in Proposition 3.1, explicit solutions of $C^A_{TB}(0)$ and $C^A_{BC}(0)$ are given as follows.

Corollary 3.2. Given initial condition of non-default and Theorem 2.2, the explicit solutions of firm A's tax benefits and bankruptcy cost respectively are the following,

$$\begin{aligned} C^A_{TB}(0) = {} & \int_0^T \beta\ c^A\,\mathrm{e}^{-rt}\left(\mathrm{N}[n_1(t)]-e^{-\alpha x_A}\mathrm{N}[n_2(t)]\right)\left(\mathrm{N}[n_5(t)]-e^{-bx_B}\mathrm{N}[n_6(t)]\right)dt \\ & + \int_0^T\int_0^t\int_{F^A+d}^{\infty}\int_{F^A}^{x}\beta\ c^A\ \mathrm{e}^{-rt}\left(\mathrm{N}[n_3(t,u,x)]-e^{x_A c(x)}\mathrm{N}[n_4(t,u,x)]\right)f_{\tau^B}(u\,|\,\mathcal{F}_0) \\ & \times f_{V,\inf \mathrm{V}}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{u\le t\})dy\ \ dx\ \ du\ \ dt \end{aligned}$$

and

$$\begin{aligned} C^A_{BC}(0) = {} & \alpha^A\Big(\int_0^T\int_{F^A}^{F^A+d}\int_{F^A}^{x}\left(x-F^A\right)f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{t\le T\})f_{\tau^B}(t\,|\,\mathcal{F}_0)\mathrm{e}^{-rt}dy\ dx\ dt \\ & + \int_0^T\int_{F^A}^{\infty}\int_{F^A+d}^{x}\int_{t_\perp}^{T} d\ \ \mathrm{e}^{-rs}f_{\tau^A}\left(s\,|\,\mathcal{F}^A_t\vee\mathcal{F}^B_0\vee\mathcal{J}_T\vee\left\{V^A(t)=x\right\}\right) \\ & \times f_{V,\inf \mathrm{V}}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{t\le T\})f_{\tau^B}(t\,|\,F_0)ds\ \ dy\ \ dx\ \ dt \\ & + \int_0^T F^A f_{\tau^A}(t\,|\,\mathcal{F}_0))\ \ \mathrm{e}^{-rt}dt) \end{aligned}$$

Using Proposition 3.1 and Corollary 3.2, the lower good-deal bound of A's total levered value equaling asset value plus tax benefits minus bankruptcy costs:

$$\underline{TV}^A(0) = V^A(0) + \underline{C}^A_{TB}(0) - \underline{C}^A_{BC}(0) \tag{3.4}$$

and equity value equaling firm's total value minus debt:

$$\begin{aligned} \underline{C}^A_{Equity}\ (0) &= \underline{TV}^A(0) - \underline{C}^A_{Debt}(0) \\ &= V^A(0) + \left(\beta-1\right)\ \underline{C1}^A_{Debt}(0) - \underline{C2}^A_{Debt}(0) - \underline{C3}^A_{Debt}(0)\,/\left(1-\alpha^A\right) \end{aligned} \tag{3.5}$$

For the objective of maximizing equity value, one can differentiate Eq. (3.5) with respect to debt's par, set this expression equal to zero, and then solve the optimal debt ratio.

4. Numerical investigation

Using the proposed structural model, this section gives a three-stage numerical analysis. In the first stage, counterparty effects on firm's optimal capital structure are examined. The question of how market incompleteness affects the counterparty effect on optimal capital structure will be explored in the next stage. The final stage intends to test the sensitivities of counterparty effects to some model's parameters strongly associated with capital structure choice, such as bankruptcy costs and tax rate. At the beginning of analysis, we must choose a set of baseline parameters to facilitate the following discussion:

$$\begin{aligned} & \mathcal{A}^2 = 10\%, \mu^A_V = \mu^B_V = \mu^A_S = \mu^B_S = 10\%, \delta^A_V = \delta^B_V = \delta^A_S = \delta^B_S = 3.75\%, \\ & \sigma_{V^A_Z} = \sigma_{V^B_Z} = 25\%, \sigma_{V^A_W} = \sigma_{V^B_W} = 35\%, \sigma_{S^A} = \sigma_{S^B} = 30\%, r = 5\%, \beta = 35\%, \\ & \alpha^A = 50\%, d\,/\,F^B = 30\%, F^B = \$20, V^A = V^B = \$100, T = 5, m = 10, a = 1. \end{aligned} \tag{4.1}$$

These values are fairly standard and broadly consistent with empirical estimates of Ju et al. (2005) and choices by Hung and Liu (2005).

4.1. Counterparty effects on optimal capital structure and underleveraged puzzle

Table 1 compiles the model outputs at optimal leverage with different levels of counterparty risk intensity, measured as the ratio of d/F^B. Firstly note from the table that optimal debt ratio (Row 1) and debt issuing amount (Row 2) both clearly have negative relation with counterparty risk intensity. The numbers respectively fall from 15.225% and \$15.35 to 14.538% and \$14.64 varying the levels of risk intensity from 0 to 50%. Since the effects of counterparty risk increase primary firm's default probabilities (Row 8) and bankruptcy costs (Row 7) but reduce the value of tax shield (Row 6), the tradeoff benefits on leverage usage become smaller with the involvement of counterparty defaults. As a result, the optimal debt ratios estimated by our model are consistently lower than that by standard model (when letting $d/F^B = 0\%$). This also indirectly helps explain the so-called under-leverage puzzle using the arguments of counterparty effects. The results on yield spread (Row 5) and coupon rate (Row 4) are not surprising, which reveal a negative correlation to counterparty risk intensity.

4.2. Does market incompleteness affect the counterparty effects on optimal leverage?

Now move the attention to Table 2 that presents the numbers of optimal leverage with different levels of the counterparty risk intensity and market incompleteness. For convenience in the discussion, we use $\sigma_{V^A_W}$ as a measure of market incompleteness. The reason is that a higher value of $\sigma_{V^A_W}$ implies a weaker correlation between non-traded assets and its twin securities, and hence, interprets a stronger degree of market incompleteness. From the

Table 1
Model outputs at optimal leverage with different levels of counterparty risk intensity.

Variables	Counterparty risk intensity (d/F^B)					
	0%	10%	20%	30%	40%	50%
Debt ratio	15.225%	15.096%	14.957%	14.815%	14.674%	14.538%
Debt's par	\$15.350	\$15.218	\$15.075	\$14.928	\$14.782	\$14.640
Coupon	\$0.889	\$0.881	\$0.871	\$0.861	\$0.850	\$0.838
Coupon rate	5.794%	5.788%	5.779%	5.765%	5.748%	5.727%
Yield spreads	79.364 bps	78.793 bps	77.856 bps	76.536 bps	74.824 bps	72.713 bps
Tax benefits	\$1.349	\$1.337	\$1.322	\$1.306	\$1.290	\$1.273
Bankruptcy costs	\$0.528	\$0.530	\$0.535	\$0.543	\$0.555	\$0.570
Default prob.	8.267%	8.292%	8.326%	8.369%	8.423%	8.488%

This table reports model outputs at optimal leverage varying the level of counterparty risk intensity from 10 to 50%. Financial variables covered by the table include optimal debt ratio (Row 1), optimal debt issuing amount (Row 2), coupon level (Row 3), annualized coupon rate (Row 4), debt's yield spreads (Row 5), the value of tax benefits (Row 6), bankruptcy costs (Row 7), and firm's default probabilities (Row 8). This table is based on the baseline parameters given in Eq. (4.1).

Table 2
Optimal leverage ratio with different levels of counterparty risk intensity and market incompleteness.

Variables	Counterparty risk intensity (d/F^B)					
	0%	10%	20%	30%	40%	50%
Panel A: $\sigma_{V_W}=15\%$(low degree of market incompleteness)						
Debt ratio	27.258%	27.127%	26.979%	26.817%	26.643%	26.463%
Debt's par	$27.715	$27.578	$27.423	$27.252	$27.069	$26.878
Yield spreads	55.947 bps	55.746 bps	55.432 bps	54.975 bps	54.365 bps	53.600 bps
Panel B: $\sigma_{V_W}=25\%$						
Debt ratio	21.164%	21.031%	20.888%	20.733%	20.574%	20.414%
Debt's par	$21.423	$21.286	$21.137	$20.976	$20.810	$20.641
Yield spreads	66.792 bps	66.444 bps	65.931 bps	65.187 bps	64.224 bps	63.021 bps
Panel C: $\sigma_{V_W}=35\%$						
Debt ratio	15.225%	15.096%	14.957%	14.815%	14.674%	14.538%
Debt's par	$15.350	$15.218	$15.075	$14.928	$14.782	$14.640
Yield spreads	79.364 bps	78.793 bps	77.856 bps	76.536 bps	74.824 bps	72.713 bps
Panel D: $\sigma_{V_W}=45\%$						
Debt ratio	10.373%	10.241%	10.111%	9.986%	9.870%	9.764%
Debt's par	$10.427	$10.293	$10.160	$10.031	$9.912	$9.802
Yield spreads	92.286 bps	91.152 bps	89.367 bps	86.836 bps	83.597 bps	79.618 bps
Panel E: $\sigma_{V_W}=55\%$						
Debt ratio	6.750%	6.621%	6.505%	6.406%	6.321%	6.255%
Debt's par	$6.771	$6.641	$6.523	$6.421	$6.334	$6.265
Yield spreads	104.687 bps	102.519 bps	98.907 bps	93.832 bps	87.340 bps	79.652 bps
Panel F: $\sigma_{V_W}=65\%$ (High degree of market incompleteness)						
Debt ratio	4.210%	4.087%	3.996%	3.927%	3.880%	3.850%
Debt's par	$4.218	$4.094	$4.001	$3.931	$3.882	$3.851
Yield spreads	116.287 bps	111.866 bps	104.240 bps	93. 529 bps	80.313 bps	65.151 bps

This table reports the optimal leverage ratio varying the levels of counterparty risk intensity (from 10 to 50%) and market incompleteness (from 15 to 65%). This table is based on the baseline parameters given in Eq. (4.1).

table, observe that the decrements in leverage-use resulted from counterparty risk are decreasing with market incompleteness. Taking the comparison among Panels A, C and F as an example, the distance in leverage ratio equals to 0.664% given low incompleteness-degree at $\sigma_{V_W}=15\%$; while the distance reduces to 0.558% and to 0.237% respectively given a median and a high degree of market incompleteness at $\sigma_{V_W}=35\%$ and $\sigma_{V_W}=65\%$. However, these seem to be weaker than one might expect. It is also observable that the market incompleteness greatly has negative impacts on optimal leverage. Optimal ratios are about 26% choosing σ_{V_W} to be 15%; while those become about 4% if letting σ_{V_W} be 65%. Such a finding is in line with Rajan and Zingales (1995) and Ju et al. (2005), due to the fact that the rise in σ_{V_W} will lead to a higher level of aggregate asset risk.

4.3. The sensitivities of counterparty effects on optimal leverage to model parameters

This subsection examines the sensitivities of counterparty effects on primary firm's optimal leverage to two model parameters strongly associated with capital structure; i.e., tax rate and bankruptcy cost rate. To do so, we firstly compute the optimal debt ratios and debt issuing amounts using standard model and ours. The differences in

Table 3
The sensitivities of counterparty effects on optimal leverage ratio to firm's tax rate and bankruptcy cost parameter.

Variables	Tax rate (β)				
	15%	25%	35%	45%	55%
Debt ratio w. counter. risk	8.814%	11.785%	14.815%	18.185%	22.123%
Debt's par w. counter. risk	$8.831	$11.837	$14.928	$18.402	$22.511
Debt ratio w/o. counter. risk	9.377%	12.263%	15.225%	18.53%	22.384%
Debt's par w/o. counter. risk	$9.398	$12.322	$15.350	$18.764	$22.794
Difference in debt ratio	−0.563%	−0.478%	−0.410%	−0.345%	−0.261%
Difference in debt's par	$−0.567	$−0.485	$−0.422	$−0.362	$−0.283
Variables	Bankruptcy cost parameter (α^A)				
	10%	20%	30%	40%	50%
Debt ratio w. counter. risk	31.746%	23.337%	19.096%	16.541%	14.815%
Debt's par w. counter. risk	$32.241	$23.609	$19.281	$16.681	$14.928
Debt ratio w/o. counter. risk	31.812%	23.583%	19.428%	16.922%	15.225%
Debt's par w/o. counter. risk	$32.322	$23.869	$19.626	$17.075	$15.350
Difference in debt ratio	−0.066%	−0.246%	−0.332%	−0.381%	−0.410%
Difference in debt's par	$−0.081	$−0.260	$−0.345	$−0.394	$−0.422

This table reports the sensitivities of counterparty effects on firm's optimal leverage ratio to tax rate (from 15 to 55%) and bankruptcy cost rate (from 10 to 50%). This table is based on the baseline parameters given in Eq. (4.1).

optimal ratio and in debt level between two models then are used to measure the intensity of counterparty effect. The results on numerical computation are summarized in Table 3.

Now put the focus on the last two rows of each panel in the table. It shows that the counterparty effects display asymmetric sensitivities. Specifically, the effects become weaker given a greater tax rate but stronger given a higher bankruptcy cost rate. By varying the tax rate from 15% to 55%, the distance in optimal debt ratio and issuing amount respectively falls from 0.563% and \$0.567 to 0.261% and \$0.283. Along with bankruptcy cost rate, these two distances are observed to rise from 0.066% and \$0.081 (when $\alpha^A = 10\%$) to 0.41% and \$0.422 (when $\alpha^A = 50\%$). The reasons behind this fact can be interpreted in two dimensions. Firstly, the rate of bankruptcy cost and tax delivers reverse impacts on the optimal capital structure. Secondly, the involvement of counterparty risk into tradeoff analysis enhances the sensitivities of capital structure to firm's characteristics.

5. Conclusion

This paper develops a static contingent-claim model that allows for examining optimal capital structure with joint arguments of market incompleteness and counter-party risk. To capture the effects of counterparty's default on contingent claims' value, a first-passage model with jump default barrier is adopted. Under primary–secondary framework, the jump in primary firm's bankruptcy triggering threshold reflects the capital loss resulted from secondary firm's default. The pricing formula of corporate debt is derived using "good-deal asset price bound" method that particularly accounts for the relevance of market incompleteness in the trading economy. The optimization of capital structure is based on the tradeoff between tax benefits and bankruptcy costs.

The numerical results elucidate the significance of counterparty default risk for determining firms' optimal financing policies. If the counterparty effects are ignored, default risk on the debt will be under-predicted such that bankruptcy costs are under-estimated and the over-use of leverage appears. The relevance of counterparty effects in the model naturally helps explain the so-called under-leveraged puzzle. We further show that the effects of counterparty default on optimal leverage ratio display clear sensitivities to market incompleteness and variables strongly associated with capital structure. Specifically, counterparty effects on capital structure become weaker given a greater tax rate and a higher degree of market incompleteness, but stronger as the rise in bankruptcy cost rate.

Our idea can still be implemented in several dimensions. The pricing of credit derivatives or of other defaultable claims can be studied. More difficult extensions include the intersection between multi-parties or a stochastic term structure of interest rate. We hope that more following papers and improvements pertaining to this subject will be devoted.

Appendix A

Proof of Theorem 2.2. To achieve the proof, we firstly introduce a useful lemma and theorem that describe the probability law of first passage time.

Lemma A1. Let τ stand for the first passage time to zero by the process Y, that is, $\tau := \inf(t \in [0, T]: Y(t) = 0)$. The process Y follows one-dimensional standard Brownian motion defined on $(\Omega, \mathcal{F}, \mathrm{Q})$ with the volatility $\sigma > 0$ and the drift $\nu \in \Re$, specifically: $dY(t) = \nu dt + \sigma dB_{\mathrm{Q}}(t)$, for $Y(0) = y(0)$. Then the random variable τ has an inverse Gaussian probability distribution under Q. For any $0 \le t \le T$,

$$\mathrm{P_Q}(\tau \le t) = \mathrm{N}[h_1(t)] + \mathrm{e}^{-2\nu\sigma^{-2}y(0)}\ \mathrm{N}[h_2(t)],$$

where $h_1(t) = [-y(0) - \nu t] / (\sigma\sqrt{t})$, and $h_2(t) = [-y(0) + \nu\ t] / (\sigma\sqrt{t})$.

Proof of Lemma A1. Note that, for any $0 \le t \le T$,

$$\mathrm{P_Q}(\tau > \mathrm{t}) = \mathrm{P_Q}\left(\inf_{0 \le s \le t} Y(s) \ge 0\right) = \mathrm{P_Q}\left(\inf_{0 \le s \le t} X(s) \ge -y(0)\right), \tag{A1}$$

where $X(s) := \nu s + \sigma B_{\mathrm{Q}}(s)$. Recall that for every $x < 0$ we have (see Corollary B.3.4 in Musiela and Rutkowski, 1997)

$$\mathrm{P_Q}\left(\inf_{0 \le s \le t} X(s) \ge x\right) = \mathrm{N}\left[(-x + \nu t)\left(\sigma\sqrt{t}\right)^{-1}\right] - \mathrm{e}^{2\nu\sigma^{-2}x}\ \mathrm{N}\left[(x + \nu t)\left(\sigma\sqrt{t}\right)^{-1}\right].$$

When combined with Eq. (A1), this yields the Lemma A1.

Theorem A2. Denote by $M(t)$ the maximum of standard Brownian motion Y on $[0, t]$. The joint distribution of $(M(t), Y(t))$ has the density, for any $y \ge x \ge 0$,

$$\begin{aligned} f_{Y,M}(x, y) &= -\partial^2 \mathrm{P_Q}(Y(t) \le x, M(t) \ge y) / \partial x \partial y \\ &= -\partial^2 \left\{1 - \mathrm{N}\left[(2y - x) / t^{1/2}\right]\right\} / \partial x \partial y \\ &= \left[(2y - x) / t^{3/2}\right] \sqrt{2/\pi} \exp\left[-(2y - x)^2 / (2t)\right]. \end{aligned}$$

Proof of Theorem A2. See Theorem 3.21 in Klebaner (2004).

By the use of famous Bayesian theorem in solving survival probability of τ^A under R on $\mathcal{F}_0 = \mathcal{F}_0^A \vee \mathcal{F}_0^B$, we have

$$\begin{aligned} \mathrm{P_R}\left(\tau^A > t \,|\, \mathcal{F}_0\right) &= \mathrm{P_R}\left(\tau^A > t \,|\, \mathcal{F}_0 \vee \left\{\tau^B > t\right\}\right) \mathrm{P_R}\left(\tau^B > t \,|\, \mathcal{F}_0\right) \\ &\quad + \mathrm{P_R}\left(\tau^A > t \,|\, \mathcal{F}_0 \vee \left\{\tau^B \le t\right\}\right) \mathrm{P_R}\left(\tau^B \le t \,|\, \mathcal{F}_0\right) \\ &:= I_1(t) I_3(t) + I_2(t)[1 - I_3(t)]. \end{aligned} \tag{A2}$$

Firstly examine I_1 to yield

$$\begin{aligned} I_1(t) &:= \mathrm{P_R}\left(\tau^A > t \,|\, \mathcal{F}_0 \vee \left\{\tau^B > t\right\}\right) \\ &= 1 - \mathrm{P_R}\left(\tau^A \le t \,|\, \mathcal{F}_0 \vee \left\{\tau^B > t\right\}\right) \\ &= 1 - \mathrm{P_R}\left(\inf_{0 \le s \le t} V^A(s) \le F^A + 1_{(\tau^B \le s)} d \,|\, \mathcal{F}_0 \vee \left\{\tau^B > t\right\}\right) \\ &= 1 - \mathrm{P_R}\left(\inf_{0 \le s \le t} V^A(s) \le F^A \,|\, \mathcal{F}_0 \vee \left\{\tau^B > t\right\}\right). \end{aligned}$$

Using the Lemma A1 with $y(0) = \ln(V^A(0)/F^A)$, $\nu = \lambda_A$, and $\sigma = \sigma_{V^A}$ gives

$$\begin{aligned} I_1(t) &= \mathrm{N}\left(\frac{\ln\left(V^A(0)/F^A\right) + \lambda_A\ t}{\sigma_{V^A}\sqrt{t}}\right) - \mathrm{e}^{2\lambda_A\left[\ln F^A - \ln V^A(0)\right]\sigma_{V^A}^{-2}} \mathrm{N}\left(\frac{\ln\left(F^A/V^A(0)\right) + \lambda_A\ t}{\sigma_{V^A}\sqrt{t}}\right) \\ &= \mathrm{N}[n_1(t)] - \left(\frac{F^A}{V^A(0)}\right)^{x_A} \mathrm{N}[n_2(t)]. \end{aligned} \tag{A3}$$

Now turn the attention on solving I_2, by standard calculations,

$$\begin{aligned} I_2(t) &:= \mathrm{P_R}\left(\tau^A > t \,|\, \mathcal{F}_0 \vee \left\{\tau^B \le t\right\}\right) \\ &= \mathrm{P_R}\left(\inf_{0 \le s \le t} V^A(s) \ge F^A + 1_{(\tau^B \le s)} d \,|\, \mathcal{F}_0 \vee \left\{\tau^B \le t\right\}\right) \\ &= \mathrm{P_R}\left(\inf_{0 \le s < \tau^B} V^A(s) \ge F^A, \inf_{\tau^B \le s \le t} V^A(s) \ge F^A + d \,|\, \mathcal{F}_0 \vee \left\{\tau^B \le t\right\}\right) \\ &= \mathrm{E_R}\left(1_{\theta_1} 1_{\theta_2} \,|\, \mathcal{F}_0 \vee \left\{\tau^B \le t\right\}\right) \end{aligned} \tag{A4}$$

where $\theta_1 := \{\omega : \inf_{0 \le s < \tau^B(\omega)} V^A(s, \omega) \ge F^A\}$ and $\theta_2 := \{\omega : \inf_{\tau^B(\omega) \le s \le t} V^A(s, \omega) \ge F^A + d\}$.

For convenience in the derivation, we introduce a right-continuous jump process

$$J(t) = 1_{(\tau^B \le t)}, \text{ for } t \in [0, \infty).$$

Thus the process J generates information set $(\mathcal{J}_t)_{t\geq 0}=J\subset\mathcal{F}^{\mathcal{A}}\vee\mathcal{F}^{\mathcal{B}}$ associated with the observation of $\tau^{\mathcal{B}}$ at continuous time. Following the iterated expectation theorem, Eq. (A4) becomes

$$\mathrm{E_R}\left(\mathrm{E_R}\left(1_{\theta_1}\mathrm{P_R}\left(\theta_2\,|\,\mathcal{F}^{\mathcal{A}}_{\tau^B}\vee\mathcal{F}^{\mathcal{B}}_0\vee\mathcal{J}_t\vee\left\{\tau^B\leq t\right\}\right)|\,\mathcal{F}_0\vee\mathcal{J}_t\vee\left\{\tau^B\leq t\right\}\right)|\,\mathcal{F}_0\vee\left\{\tau^B\leq t\right\}\right). \tag{A5}$$

By the use of Lemma A1 with $y(0)=\ln(V^A(\tau^B)/(F^A+d))$, $\nu=\lambda_A$, and $\sigma=\sigma_{V^A}$, we solve $\mathrm{P_R}(\theta_2|\mathcal{F}^{\mathcal{A}}_{\tau^B}\vee\mathcal{F}^{\mathcal{B}}_0\vee\mathcal{J}_t\vee\{\tau^B\leq t\})\coloneqq I_{2A}(t;V^A(\tau^B),\tau^B)$ as below.

$$\begin{aligned}I_{2A}\left(t;\tau^B=u,V^A(\tau^B)=x\right)&=\mathrm{N}\left(\frac{\ln x-\ln\left(F^A+d\right)+\lambda_A(t-u)}{\sigma_{V^A}\sqrt{t-u}}\right)\\&-\mathrm{N}\left(\frac{\ln\left(F^A+d\right)-\ln x+\lambda_A(t-u)}{\sigma_{V^A}\sqrt{t-u}}\right)\mathrm{e}^{2\lambda_A\left[\ln\left(F^A+d\right)-\ln x\right]\sigma_{V^A}^{-2}}\\&=\mathrm{N}[n_3(t,u,x)]-\mathrm{N}[n_4(t,u,x)]\left(\frac{F^A+d}{x}\right)^{x_A}.\end{aligned} \tag{A6}$$

Note that, on $\mathcal{F}_0\vee\mathcal{J}_t\vee\{\tau^B\leq t\}$, terms $\inf_{0\leq s<\tau^B}V^A(s)$ and $V^A(\tau^B)$ are immeasurable only. To have the density of this joint distribution, by Theorem A2 and reflection principle, for any $x\geq y\geq 0$,

$$\begin{aligned}&f_{V,\inf V}\left(V^A(u)=x,\inf_{0\leq s<u}V^A(s)=y\,|\,\mathcal{F}_0\vee\mathcal{J}_t\vee\left\{\tau^B=u\leq t\right\}\right)\\&=-\frac{\partial^2\mathrm{P_R}\left(\inf_{0\leq s<u}V^A(s)\leq y,\ V^A(u)\geq x\,|\,\mathcal{F}_0\vee\mathcal{J}_t\vee\left\{\tau^B=u\leq t\right\}\right)}{\partial x\partial y}\\&=-\frac{\partial^2\left\{\mathrm{N}\left[2\ln y-\ln x-\ln V^A(0)+\lambda_A u/\left(\sigma_{V^A}\sqrt{u}\right)\right]\left[y/V^A(0)\right]^{2\lambda_A\sigma_{V^A}^{-2}}\right\}}{\partial x\partial y}\\&=\frac{\sqrt{2}\ln\left(xV^A(0)/y^2\right)}{\sqrt{\pi}\ u^{3/2}\sigma^3_{V^A}\ xy}\exp\left(\frac{\left(2\ln y-\ln x-\ln V^A(0)+\lambda_A u\right)^2}{-2\ \sigma^2_{V^A}\ u}\right)\left(\frac{y}{V^A(0)}\right)^{x_A}\end{aligned} \tag{A7}$$

Now substitute Eqs. (A6) and (A7) into Eq. (A5) to rewrite the conditional expectation. Since $\theta_1:=\{\omega:\inf_{0\leq s<\tau^B(\omega)}V^A(s,\omega)\geq F^A\}$ and $\theta_2\supset\{\omega:V^A(\tau^B,\omega)\geq F^A+d\}$, we have

$$\mathrm{E_R}\left(\int_{F^A+d}^{\infty}\int_{F^A}^{x}I_{2A}(t;u,x)f_{V,\inf V}\left(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_t\vee\left\{\tau^B=u\leq t\right\}\right)dy\,dx\,|\,\mathcal{F}_0\vee\left\{\tau^B\leq t\right\}\right). \tag{A8}$$

To explicitly solve Eq. (A8), examining the density function of τ^B on $\mathcal{F}_0\vee\{\tau^B\leq t\}$ under R is required. That is, for $u\leq t$,

$$f_{\tau^B}\left(\tau^B=u\,|\,\mathcal{F}_0\vee\{u\leq t\}\right)=\partial\mathrm{P_R}\left(\tau^B\leq u\,|\,\mathcal{F}_0\vee\left\{\tau^B\leq t\right\}\right)/\partial u$$

where

$$\begin{aligned}\mathrm{P_R}\left(\tau^B\leq u\,|\,\mathcal{F}_0\vee\left\{\tau^B\leq t\right\}\right)&=\mathrm{P_R}\left(\tau^B\leq u,\tau^B\leq t\,|\,\mathcal{F}_0\right)/\mathrm{P_R}\left(\tau^B\leq t\,|\,\mathcal{F}_0\right)\\&=\mathrm{P_R}\left(\tau^B\leq u\,|\,\mathcal{F}_0\right)/\mathrm{P_R}\left(\tau^B\leq t\,|\,\mathcal{F}_0\right)\\&=[1-I_3(u)][1-I_3(t)]^{-1}.\end{aligned}$$

Again using Lemma A1 with $y(0)=\ln(V^B(0)/F^B)$, $\nu=\lambda_B$, and $\sigma=\sigma_{V^B}$ yields

$$\begin{aligned}I_3(t)&=\mathrm{N}\left(\frac{\ln\left(V^B(0)/F^B\right)+\lambda_B t}{\sigma_{V^B}\sqrt{t}}\right)-\left(\frac{F^B}{V^B(0)}\right)^{x_B}\mathrm{N}\left(\frac{\ln\left(F^B/V^B(0)\right)+\lambda_B t}{\sigma_S\sqrt{t}}\right)\\&=\mathrm{N}[n_5(t)]-\left(\frac{F^B}{V^B(0)}\right)^{x_B}\mathrm{N}[n_6(t)].\end{aligned} \tag{A9}$$

Given Eq. (A9), the density function of $\tau^B=u$ on $\mathcal{F}_0\vee\{\tau^B\leq t\}$ under R is

$$\begin{aligned}f_{\tau^B}(u\,|\,\mathcal{F}_0\vee\{u\leq t\})&=\left\{\left(F^B/V^B(0)\right)^{x_B}(2\pi)^{-0.5}\exp\left(-n_6(u)^2 2^{-1}\right)\left[2\lambda_B-u^{-1}(-b+\lambda_B u)\right]\right.\\&\times(2\sigma_{V^B}\sqrt{u})^{-1}-(2\pi)^{-0.5}\exp\left(-n_5(u)^2 2^{-1}\right)\left[2\lambda_B-u^{-1}(b+\lambda_B u)\right]\\&\left.\times(2\sigma_{V^B}\sqrt{u})^{-1}\right\}\left\{1-\mathrm{N}[n_5(t)]+\left(F^B/V^B(0)\right)^{x_B}\mathrm{N}[n_6(t)]\right\}^{-1}\\&=f_{\tau^B}(u|F_0)/(1-I_3(t)).\end{aligned} \tag{A10}$$

When Eq. (A8)is combined with Eq. (A10), we have

$$I_2(t)=\int_0^t\int_{F^A+d}^{\infty}\int_{F^A}^{x}I_{2A}(t;u,x)f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_t\square\{u\leq t\})\ f_{\tau^B}(u\,|\,\mathcal{F}_0\vee\{u\leq t\})dy\ dx\ du. \tag{A11}$$

Lastly, substituting Eqs. (A3), (A6), (A7) and (A9)–(A11) into Eq. (A2) completes the proof.

Proof of Proposition 3.1. Since the detailed derivation of $F_{\tau^A}(\cdot)$ is given, the proof here mainly focuses on solving $\mathrm{E_R}(V^A(\tau^A)\mathrm{e}^{-r\tau^A}1_{(\tau^A\leq T)}|\mathcal{F}_0)$. It is known from Eq. (2.2) that, at default time, the value of A's remaining asset is $V^A(\tau^A)=F^A+1_{(\tau^A>\tau^B)}d+1_{(\tau^A=\tau^B)}\varphi$ where φ is a $\mathcal{F}^{\mathcal{A}}_{\tau^B}$-measurable variable from Ω to $[0,d]$. Hence, we have

$$\begin{aligned}I_4&\coloneqq\mathrm{E_R}\left(V^A\left(\tau^A\right)\mathrm{e}^{-r\tau^A}1_{(\tau^A\leq T)}\,|\,\mathcal{F}_0\right)\\&=\mathrm{E_R}\left(F^A\mathrm{e}^{-r\tau^A}1_{(\tau^A\leq T)}\,|\,\mathcal{F}_0\right)+\mathrm{E_R}\left(d\mathrm{e}^{-r\tau^A}1_{(\tau^B<\tau^A\leq T)}\,|\,\mathcal{F}_0\right)+\mathrm{E_R}\left(\varphi\mathrm{e}^{-r\tau^A}1_{(\tau^B=\tau^A\leq T)}\,|\,\mathcal{F}_0\right)\\&\coloneqq I_{4A}+I_{4B}+I_{4C}\end{aligned} \tag{A12}$$

Given the density function of τ^A on F_0 under R, denoted by f_{τ^A}, we firstly solve I_{4A} as the following

$$I_{4A}=\int_0^T F^A f_{\tau^A}(t\,|\,\mathcal{F}_0)\mathrm{e}^{-rt}dt \tag{A13}$$

where $f_{\tau^A}(t|\mathcal{F}_0)=-\partial\,F_{\tau^A}(t|\mathcal{F}_0)/\partial t$.

Now turn the attention to solve I_{4B}. Twice using iterated expectation theorem has

$$I_{4B}=\mathrm{E_R}\left(\mathrm{E_R}\left(\mathrm{E_R}\left(d\mathrm{e}^{-r\tau^A}1_{(\tau^B<\tau^A\leq T)}\,|\,\mathcal{F}^{\mathcal{A}}_{\tau^B}\vee\mathcal{F}^{\mathcal{B}}_0\vee\mathcal{J}_T\right)|\,\mathcal{F}_0\vee\mathcal{J}_T\right)|\,\mathcal{F}_0\right). \tag{A14}$$

Note that

$$1_{(\tau^B<\tau^A\leq T)}=1_{(\tau^B\leq T)}1_{(\tau^A\leq T)}1_{(\tau^B<\tau^A)},$$

where

$$\begin{aligned}\left\{\omega:\tau^B(\omega)<\tau^A(\omega)\leq T\right\}\equiv&\left\{\omega:\inf_{0\leq s\leq\tau^B}V^A(s,\omega)\geq F^A,V^A\left(\tau^B,\omega\right)\in\left[F^A+d,\infty\right]\right\}\\&\cap\left\{\omega:\inf_{\tau^B<s\leq T}V^A(s,\omega)\leq F^A+d\right\}\cap\left\{\omega:\inf_{0\leq s<T}V^B(s,\omega)\leq F^B\right\}.\end{aligned}$$

Because τ^B, $\inf_{0\leq s<\tau^B}V^A(s)$, and $V^A(\tau^B)$ all are measurable on $\mathcal{F}^{\mathcal{A}}_{\tau^B}\vee\mathcal{F}^{\mathcal{B}}_0\vee\mathcal{J}_T$, we can rewrite Eq. (A14) as

$$\mathrm{E_R}\left(\mathrm{E_R}\left(\mathrm{E_R}\left(d\mathrm{e}^{-r\tau^A}1_{(\tau^A\leq T)}\,|\,\mathcal{F}^{\mathcal{A}}_{\tau^B}\vee\mathcal{F}^{\mathcal{B}}_0\vee\mathcal{J}_T\right)1_{(\tau^B\leq T)}1_{(\tau^B<\tau^A)}\,|\,\mathcal{F}_0\vee\mathcal{J}_T\right)|\,\mathcal{F}_0\right). \tag{A15}$$

On $\mathcal{F}^{\mathcal{A}}_{\tau^B}\vee\mathcal{F}^{\mathcal{B}}_0\vee\mathcal{J}_T$, the density function of $\tau^A=s$ under R has the following form, for $\tau^B<s\leq T$,

$$\begin{aligned}f_{\tau^A}\left(s\,|\,\mathcal{F}^{\mathcal{A}}_{\tau^B}\vee\mathcal{F}^{\mathcal{B}}_0\vee\mathcal{J}_T\right)&\equiv f_{\tau^A}\left(s\,|\,\mathcal{F}^{\mathcal{A}}_{\tau^B}\vee\mathcal{F}^{\mathcal{B}}_0\vee\mathcal{J}_T\vee\left\{V^A\left(\tau^B\right)=x\right\}\right)\\&=-\partial\mathrm{P_R}\left(\tau^A>s\,|\,\mathcal{F}^{\mathcal{A}}_{\tau^B}\vee\mathcal{F}^{\mathcal{B}}_0\vee\mathcal{J}_T\vee\left\{V^A\left(\tau^B\right)=x\right\}\right)/\partial s\\&=\left[\left(F^A+d\right)/x\right]^{x_A}(2\pi)^{-0.5}\exp\left(-n_4\left(s,\tau^B,x\right)^2 2^{-1}\right)\left(2\sigma_{V^A}\sqrt{s-\tau^B}\right)^{-1}\\&\quad\times\left\{2\lambda_A-\left(s-\tau^B\right)^{-1}\left(\ln\left[\left(F^A+d\right)/x\right]+\lambda_A\left(s-\tau^B\right)\right)\right\}\\&\quad-(2\pi)^{-0.5}\exp\left(-n_3\left(s,\tau^B,x\right)^2 2^{-1}\right)\left(2\sigma_{V^A}\sqrt{s-\tau^B}\right)^{-1}\\&\quad\times\left\{2\lambda_A-\left(s-\tau^B\right)^{-1}\left(\ln\left[x/\left(F^A+d\right)\right]+\lambda_A\left(s-\tau^B\right)\right)\right\}\end{aligned} \tag{A16}$$

Combining Eq. (A15) with Eq. (A16) gives

$$\mathrm{E_R}(\mathrm{E_R}(1_{(\tau^B\le T)}1_{(\tau^B<\tau^A)}\int_{\tau_+^B}^{T} d\,\mathrm{e}^{-rs}f_{\tau^A}\left(s\,|\,\mathcal{F}_{\tau^B}^{\mathcal{A}}\vee\mathcal{F}_0^{\mathcal{B}}\vee\mathcal{J}_T\vee\left\{V^A\left(\tau^B\right)=x\right\}\right)ds\,|\,\mathcal{F}_0\vee\mathcal{J}_T)\,|\,\mathcal{F}_0). \tag{A17}$$

Since $\{\omega:\tau^B(\omega)<\tau^A(\omega),\ \omega\in\Omega\}\equiv\{\omega:\inf_{0\le s<\tau^B(\omega)}V^A(s,\ \omega)\ge F^A,\ V^A(\tau^B,\ \omega)\in[F^A+d,\ \infty]\}$ and τ^B is measurable on $\mathcal{F}_0\vee\mathcal{J}_T$, Eq. (A17) becomes

$$\mathrm{E_R}(1_{(\tau^B\le T)}\int_{F^A}^{\infty}{}_{+\,d}\int_{F^A}^{x}\int_{\tau_+^B}^{T} d\mathrm{e}^{-rs}f_{\tau^A}\left(s\,|\,\mathcal{F}_{\tau^B}^{\mathcal{A}}\vee\mathcal{F}_0^{\mathcal{B}}\vee\mathcal{J}_T\vee\left\{V^A\left(\tau^B\right)=x\right\}\right)$$
$$\times f_{V,\inf V}\left(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\left\{\tau^B\le T\right\}\right)ds\ dy\ dx\,|\,\mathcal{F}_0)$$

Again using the density function of τ^B on F_0 under R can yield

$$I_{4B}=\int_0^T\int_{F^A+d}^{\infty}\int_{F^A}^{x}\int_{t_+}^{T} d\ \mathrm{e}^{-rs}f_{\tau^A}\left(s\,|\,\mathcal{F}_t^{\mathcal{A}}\vee\mathcal{F}_0^{\mathcal{B}}\vee\mathcal{J}_T\vee\left\{V^A(t)=x\right\}\right)f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{t\le T\})$$
$$\times f_{\tau^B}(t\,|\,\mathcal{F}_0)ds\ dy\ dx\ dt. \tag{A18}$$

Lastly, by iterated expectation theorem, we can write I_{4C} as below

$$I_{4C}=\mathrm{E_R}\left(\mathrm{E_R}\left(\varphi 1_{\theta_3}\,|\,\mathcal{F}_0\vee\mathcal{J}_T\right)\mathrm{e}^{-r\tau^B}1_{\{\tau^B\le T\}}\,|\,\mathcal{F}_0\right)$$

where $\theta_3:=\left\{\omega:\tau^B(\omega)=\tau^A(\omega)\right\}\equiv\left\{\omega:\inf_{0\le s<\tau^B}V^A(s,\omega)\ge F^A,V^A\left(\tau^B,\omega\right)\in\left[F^A,F^A+d\right]\right\}$, and $1_{\{\tau^B=\tau^A\}}1_{\{\tau^A\le T\}}\equiv 1_{\{\tau^B=\tau^A\}}1_{\{\tau^B\le T\}}$. Since, conditioned on θ_3, the value of A's remaining assets is

$$V^A\left(\tau^A\right)=F^A+\varphi=V^A\left(\tau^B\right),$$

using the joint density function $f_{V,\inf V}(x,\ y|\mathcal{F}_0\vee\mathcal{J}_T\vee\{\tau^B\le T\})$ in the derivation gives

$$\mathrm{E_R}\left(\int_{F^A}^{F^A+d}\int_{F^A}^{x}\left(x-F^A\right)f_{V,\inf V}\left(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\left\{\tau^B\le T\right\}\right)dy\ dx\ \mathrm{e}^{-r\tau^B}1_{\{\tau^B\le T\}}\,|\,\mathcal{F}_0\right).$$

Again using the density function of τ^B on F_0 under R yields

$$I_{4C}=\int_0^T\int_{F^A}^{F^A+d}\int_{F^A}^{x}\left(x-F^A\right)f_{V,\inf V}(x,y\,|\,\mathcal{F}_0\vee\mathcal{J}_T\vee\{t\le T\})\ f_{\tau^B}(t\,|\,\mathcal{F}_0)\mathrm{e}^{-rt}dy\ dx\ dt \tag{A19}$$

Lastly, substituting Eqs. (A13), (A18) and (A19) into Eq. (A12) completes the proof.

References

Brigo, D., Tarenghi, M., 2005. Credit Default Swap Calibration and Counterparty Risk Valuation with a Scenario Based First Passage Model. Working Paper. King's College, London.

Chang, C., Yu, M., 2009. Valuing Mortgage Insurance Contracts with Counterparty Risk and Capital Forbearance. Working Paper. National Kaohsiung University of Applied Science, Taiwan.

Chen, H., 2010. Macroeconomic conditions and the puzzles of credit spreads and capital structure. J. Finance 65, 2171–2212.

Cochrane, J.H., Saa-Requejo, J., 2000. Beyond arbitrage: good-deal asset price bounds in incomplete markets. J. Political Economy 108, 79–119.

Duffie, D., Zhu, H., 2009. Does a Central Clearing Counterparty Reduce Counterparty Risk? Working Paper. Stanford University.

Eisfeldt, A.L., 2007. Smoothing with liquid and illiquid assets. J. Monetary Econ. 54, 1572–1586.

Gibson, R., Mougeot, N., 2004. The pricing of systematic liquidity risk: empirical evidence from the US stock market. J. Bank. Finance 28, 157–178.

Hackbarth, D., Miao, J., Morellec, E., 2006. Capital structure, credit risk, and macroeconomic conditions. J. Financ. Econ. 82, 519–550.

Hung, M.W., Liu, Y.H., 2005. Pricing vulnerable options in incomplete markets. J. Futures Markets 25, 135–170.

Jarrow, R.A., Turnbull, S., 1995. Pricing derivatives on financial securities subject to credit risk. J. Finance 50, 53–86.

Jarrow, R.A., Yu, F., 2001. Counterparty risk and the pricing of defaultable securities. J. Finance 56, 1765–1799.

Jensen, M.C., Meckling, W.H., 1976. Theory of the firm: managerial behavior, agency costs, and ownership structure. J. Financ. Econ. 3, 305–360.

Jorion, P., Zhang, G., 2009. Credit contagion from counterparty risk. J. Finance 64, 2053–2087.

Ju, N., Parrino, R., Poteshman, A.M., Weisbach, M.S., 2005. Horses and rabbits? Trade-off theory and optimal capital structure. J. Financ. Quantitative Anal. 40, 259–281.

Ju, N., 2001. Dynamic Optimal Capital Structure and Maturity Structure. Working Paper. University of Maryland.

Khandani, A.E., Lo, A.W., 2009. Illiquidity Premia in Asset Returns: An Empirical Analysis of Hedge Funds, Mutual Funds, and U.S. Equity Portfolios. Working Paper. Massachusetts Institute of Technology.

Klebaner, F.C., 2004. Introduction to Stochastic Calculus with Applications. Imperial College Press London: Imperial College.

Kraft, H., Steffensen, M., 2007. Bankruptcy, counterparty risk, and contagion. Rev. Finance 13, 209–252.

Leland, H.E., 1998. Agency costs, risk management, and capital structure. J. Finance 53, 1213–1243.

Leung, K.S., Kwok, Y.K., 2005. Credit default swap valuation with counterparty risk. Kyoto Econ. Rev. 74, 25–45.

Leung, K.S., Kwok, Y.K., 2009. Counterparty risk for credit default swaps: Markov chain interacting intensities model with stochastic intensity. Asia Pacific Finance Markets 16, 169–181.

Martineza, M.A., Nietob, B., Rubio, G., Tapiac, M., 2005. Asset pricing and systematic liquidity risk: an empirical investigation of the Spanish stock market. Inter. Rev. Econ. Finance 14, 81–103.

Merton, R.C., 1974. On the pricing of corporate debt: the risk structure of interest rates. J. Finance 29, 449–470.

Modigliani, F., Miller, M., 1958. The cost of capital, corporation finance, and the theory of investment. A. Econ. Rev. 48, 261–297.

Morellec, E., 2001. Asset liquidity, capital structure, and secured debt. J. Financ. Econ. 61, 173–206.

Musiela, M., Rutkowski, M., 1997. Martingale Method in Financial Modelling. Springer, Berlin Heidelberg New York.

Myers, S.C., 1977. Determinants of corporate borrowing. J. Financ. Econ. 5, 147–175.

Myers, S.C., Majluf, N.S., 1984. Corporate financing and investment decision when firms have information that investors do not have. J. Financ. Econ. 13, 187–221.

Rajan, R.G., Zingales, L., 1995. What do we really know about capital structure? Evidence from international data. J. Finance 50, 1421–1460.

Strebulaev, I., 2003. Liquidity and Asset Pricing: Evidence from the U.S. Treasury Securities Market. Working Paper. Stanford University.

Thompson, J.R., 2007. Counterparty Risk in Insurance Contracts: Should the Insured Worry About the Insurer? Working Paper. Queen's University, Canada.

Turnbull, S., 2005. The pricing implications of counterparty risk for non linear credit products. J. Credit risk 1, 3–30.

Walker, M.B., 2006. Credit default swaps with counterparty risk: a calibrated Markov model. J. Credit Risk 2, 31–49.